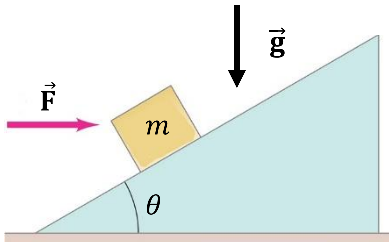
# PHYS 101 – General Physics I Midterm Exam 2

Duration: 120 minutes　　　　Saturday, 20 April 2024; 13:30

**1.** A block of mass $m$ is pushed up a distance $d$ along a frictionless inclined plane by a constant horizontal force of magnitude $F$.




(a) (6 Pts.) How much work is done by the horizontal force $\vec{\mathbf{F}}$ on the block?

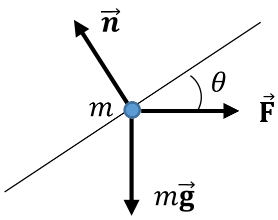
(b) (6 Pts.) How much work is done by the gravitational force on the block?

(c) (6 Pts.) How much work is done by the normal force?

(d) (7 Pts.) What is the speed of the block after this displacement? (Assume that it is zero initially.)

**Solution:** (a) Work done by a constant force $\vec{\mathbf{F}}$ in displacing an object along a straight line $\vec{\mathbf{d}}$ is

$$W_F = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} \quad \rightarrow \quad W_F = Fd\cos\theta$$




(b)

$$W_g = mgd\cos\left(\theta + \frac{\pi}{2}\right) \quad \rightarrow \quad W_g = -mgd\sin\theta$$

(c)

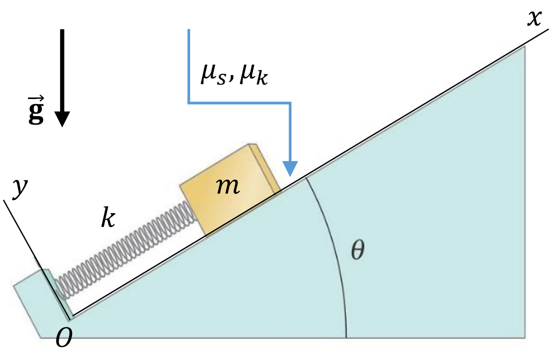
$$W_n = 0$$

(d)

$$W_{net} = \Delta K = K_f - K_i \quad \rightarrow \quad (F\cos\theta - mg\sin\theta)d = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2d}{m}(F\cos\theta - mg\sin\theta)}$$

**2.** A spring with stiffness constant $k$ and equilibrium length $L$ is placed on a slope which makes an angle $\theta$ with respect to the horizontal. One end of the spring is fixed, and a block of mass $m$ is placed (not fixed) at its free end. Coefficients of static and kinetic friction between the block and the surface on which it is placed are $\mu_s$ and $\mu_k$, respectively. Use the coordinate system indicated in the figure to answer the following questions in terms of the given quantities. (Gravitational acceleration is g.)




(a) (6 Pts.) What is the maximum compression of the spring if the mass is to remain at rest when released?

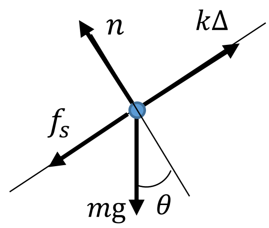
The spring is compressed to half its equilibrium length and released. Assume that $k$ is large so that the block looses contact with the spring.

(b) (6 Pts.) What will be the speed of the block at the instant it loses contact with the spring?

(c) (7 Pts.) How far up the slope from its initial position will the block move before coming to rest?

(d) (6 Pts.) What is the minimum value of the coefficient of static friction which will prevent the block from sliding back down once it stops at the top?

**Solution:** (a) Let $\Delta = L - x$ denote compression of the spring.




$$k\Delta - f_s - mg\sin\theta = 0\,, \qquad n - mg\cos\theta = 0\,, \qquad f_s \le \mu_s n$$

$$f_s = k\Delta - mg\sin\theta \le \mu_s mg\cos\theta \quad \rightarrow \quad \Delta \le \frac{mg}{k}(\sin\theta + \mu_s\cos\theta)$$

(b) Let $U_g = 0$ at the initial positon of the block. $f_k = \mu_k mg\cos\theta$

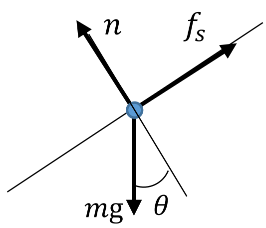
$$\Delta E = K_f + U_f - K_i - U_i = W_{f_k} \quad \rightarrow \quad \frac{1}{2}mv^2 + mg\left(\frac{L}{2}\right)\sin\theta - \frac{1}{2}k\left(\frac{L}{2}\right)^2 = -\mu_k mg\left(\frac{L}{2}\right)\cos\theta$$

$$v = \sqrt{\frac{KL^2}{4m} - gL(\sin\theta + \mu_k\cos\theta)}\,.$$

(c)

$$\Delta E = mgd\sin\theta - \frac{1}{2}k\left(\frac{L}{2}\right)^2 = -\mu_k mgd\cos\theta$$

$$d = \frac{kL^2}{8mg(\sin\theta + \mu_k\cos\theta)}\,.$$




(d)

$$f_s - mg\sin\theta = 0\,, \qquad n - mg\cos\theta = 0\,, \qquad f_s \le \mu_s n$$

$$mg\sin\theta \le \mu_s mg\cos\theta \quad \rightarrow \quad \mu_s \ge \tan\theta\,.$$

**3.** The figure illustrates three blocks on a horizontal frictionless surface, two of which are at rest. The block with mass $2m$ is fixed to a spring with stiffness constant $k$, whose other end is fixed to a wall. The block with mass $m$ moving with speed $v_0$ makes an elastic collision with the stationary block of mass $m$.

(a) (10 Pts.) What will be the velocities of the two mass $m$ blocks after the collision?

(b) (15 Pts.) Following the collision, the block with mass $m$ which was initially at rest makes a completely inelastic collision with the block of mass $2m$. Assume that the collision is instantaneous. Find the maximum compression of the spring.


**Solution:** (a)

$$p_f = p_i \quad \rightarrow \quad mv_1' + mv_2' = mv_0 \quad \rightarrow \quad v_0 - v_1' = v_2'$$

$$K_f = K_i \quad \rightarrow \quad \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 = \frac{1}{2}mv_0^2 \quad \rightarrow \quad v_2'^2 = v_0^2 - v_1'^2 \quad \rightarrow \quad v_2'^2 = (v_0 - v_1')(v_0 + v_1')$$

$$v_2' = v_0 + v_1'$$

$$v_1' = 0\,, \qquad v_2' = v_0\,.$$

(b)

$$p_f = p_i \quad \rightarrow \quad mv_0 = 3mv' \quad \rightarrow \quad v' = \frac{1}{3}v_0$$

Following the collision total mechanical energy of the system is conserved. Therefore

$$\frac{1}{2}(3m)\left(\frac{1}{3}v_0\right)^2 = \frac{1}{2}kx_m^2 \quad \rightarrow \quad x_m = \sqrt{\frac{mv_0^2}{3k}}\,.$$

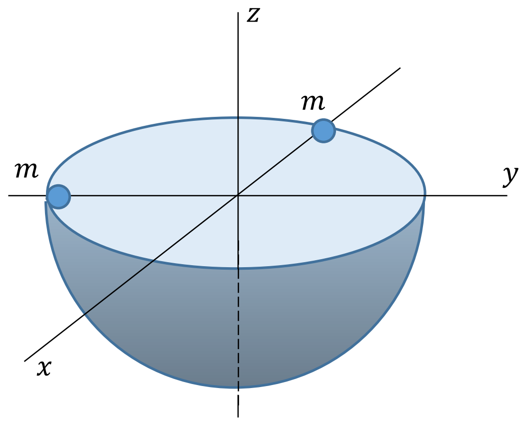
**4.** Two particles of mass $m$ are placed on the inner surface and at the edges of a frictionless hemispherical bowl of radius $R$. In the coordinate system shown in the figure, one of them is at the point $x = -R, y = 0, z = 0$, while the other is at $x = 0, y = -R, z = 0$. Gravitational acceleration is g in the negative z direction. Both are released at the same moment. They slide down and collide at the bottom of the bowl at $x = 0, y = 0, z = -R$. The collision is completely inelastic.

(a) (7 Pts.) What will be the velocity immediately after the collision?

(b) (6 Pts.) How much energy is lost in the collision?

(c) (6 Pts.) How high will the combined particle rise from the bottom of the bowl?

(d) (6 Pts.) What will be the $x$ and $y$ coordinates of the highest point the combined particle rises?




**Solution:** (a) As two particles slide down the inner surface of the frictionless hemispherical bowl their total energy will be conserved. They will have equal speeds at the bottom just before they collide.

$$\frac{1}{2}mv^2 = mgR \quad \rightarrow \quad v = \sqrt{2gR}.$$

One will be moving in the $x$ direction, while the othe will be moving in the $y$ direction. Therefore, total momentum of the system of two particles before the collision will be $p_i = m\sqrt{2gR}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$. Since momentum is conserved in the collision, we have

$$p_f = 2m\vec{\mathbf{v}}' = m\sqrt{2gR}(\hat{\mathbf{i}} + \hat{\mathbf{j}}) \quad \rightarrow \quad \vec{\mathbf{v}}' = \sqrt{\frac{gR}{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})\,.$$

(b)

$$\Delta K = \frac{1}{2}(2m)v'^2 - 2mgR \quad \rightarrow \quad \Delta K = -mgR$$

(c) Energy will be conserved following the collision. Therefore

$$\frac{1}{2}(2m)v'^2 = 2mgh \quad \rightarrow \quad h = \frac{v'^2}{2g} \quad \rightarrow \quad h = \frac{R}{2}.$$

(d) The combined particle will be moving on the inner surface of the hemisphere $x^2 + y^2 + z^2 = R^2$ such that $x = y$. Since at its highest point $z = -R/2$,

$$2x^2 + \left(\frac{R}{2}\right)^2 = R^2 \quad \rightarrow \quad x = \sqrt{\frac{3}{2}}\left(\frac{R}{2}\right), \qquad y = x = \sqrt{\frac{3}{2}}\left(\frac{R}{2}\right).$$